A Simplified Nelson-Barr Relaxion

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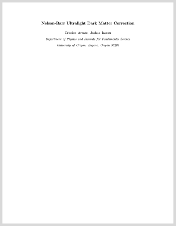
Overview

The Timeline

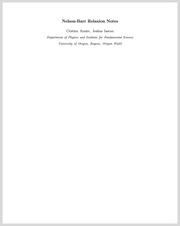


Nelson-Barr Ultralight Dark Matter Correction

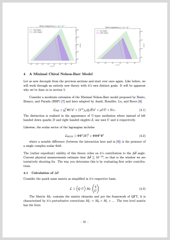
→

QFT (i-iii) and Particle (i-ii)

→

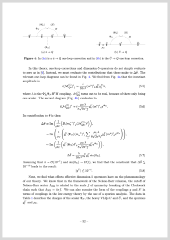


Nelson-Barr Relaxion paper replication



Minimal Chiral Nelson-Barr Relaxion

→

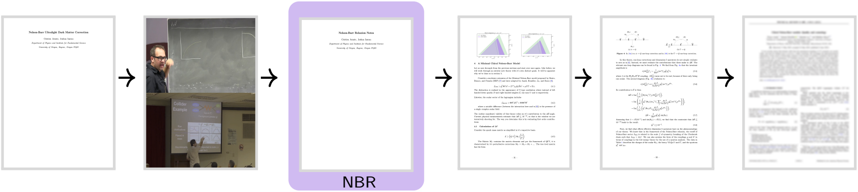


A Simplified Nelson-Barr Relaxion

→

Paper

Nelson-Barr Relaxion | Old Results



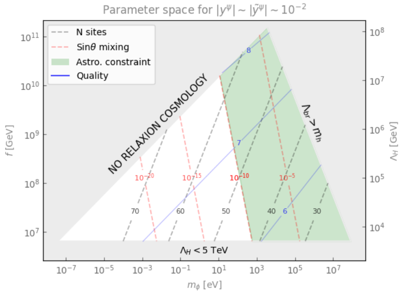


Figure 1: Our parameter space

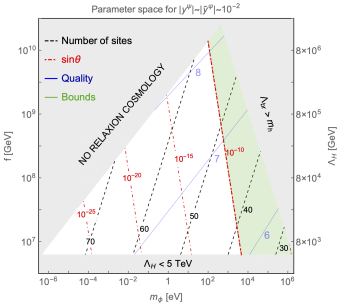
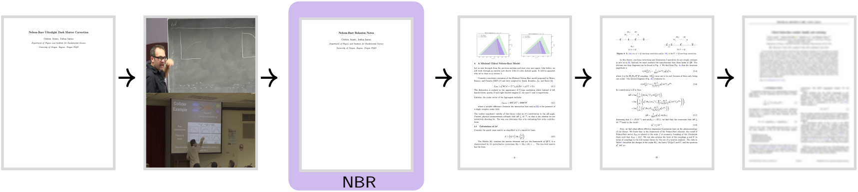


Figure 2: Davidi, Gupta, et. all [2]

Nelson-Barr Relaxion | Updated Results



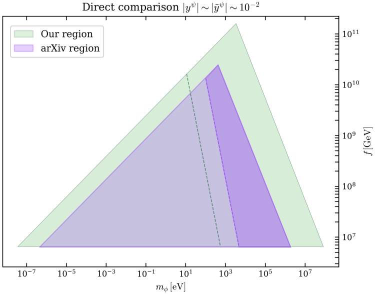


Figure 3: Parameter space comparison (old)

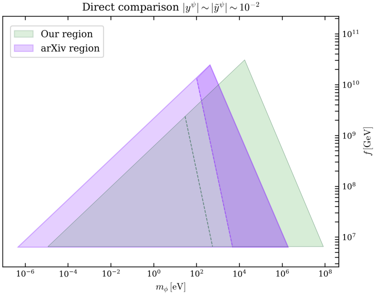
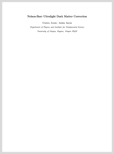
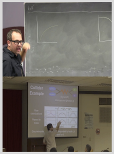
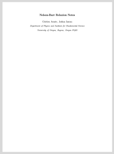
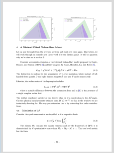
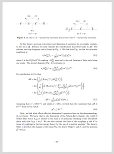


Figure 4: Parameter space comparison (new)

A Chiral Nelson-Barr Relaxion

 →  →  →  →  → 

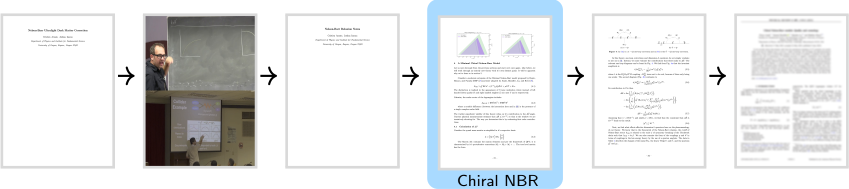
Chiral NBR

Dangerous dim-5 and dim-6 operators

	$\mathbb{Z}_2$	$U(1)_N$	$U(1)_U$	$U(1)_{\mu_U}$
Φ_N	$-$	-1	0	0
Φ_N^*	$-$	-1	0	0
U	$-$	0	1	0
$\bar{U}$	$-$	0	-1	1
y_i^U	$+$	1	-1	0
$\tilde{y}_i^U$	$+$	-1	-1	0
μ_U	$+$	0	0	-1

$$\mathcal{L}_{\mathrm{EFT}} = \frac{(\tilde{y}_j^U)^*}{\Lambda_{\mathrm{EFT}}^2}\left(y_i^U \alpha^{ij} \mu_U \Phi^2 U \bar{U} + \beta^{ij} \mu_U \Phi^* Q_i \tilde{H} \bar{U} + y_i^U \gamma^{ijk}{}_l \Phi^2 Q_k \tilde{H} \bar{u}^l\right) + \mathrm{H.c.} \qquad (1)$$

A Chiral Nelson-Barr Relaxion



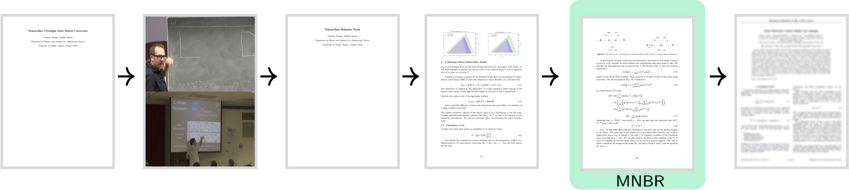
Example of vanishing contribution (no quality problem).

$$\mathcal{L}_{\rm EFT} = \frac{(\tilde{y}_j^U)^*}{\Lambda_{\rm EFT}}(y_i^U \alpha^{ij} \mu_U \Phi^2 U \bar{U}) + \text{H.c.}. \qquad (2)$$

Does not contribute to $\Delta\bar{\theta}$

$$\begin{aligned}\Delta\bar{\theta} &\sim \frac{\alpha^{ij}}{\Lambda_{\rm EFT}^2}(y_i^U \tilde{y}_j^U - y_j^U \tilde{y}_i^U)|\langle\Phi\rangle|^2 \sin 2\theta_N. \\ &\sim 0\end{aligned}$$

A Simplified Nelson-Barr Relaxion



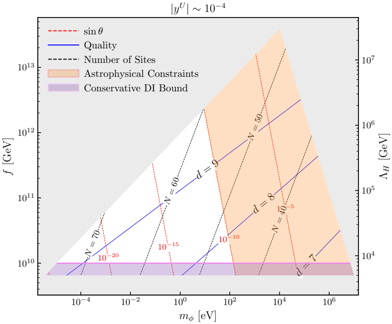


Figure 5: Conservative result

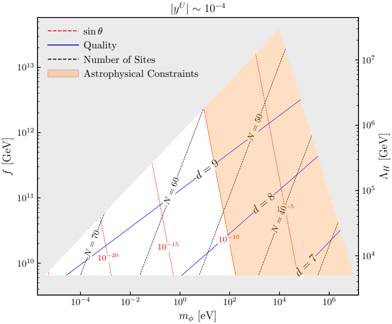
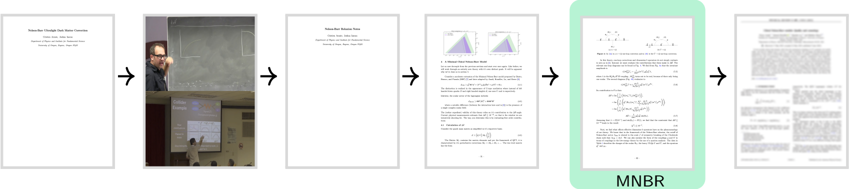


Figure 6: Optimistic result

A Simplified Nelson-Barr Relaxion Results



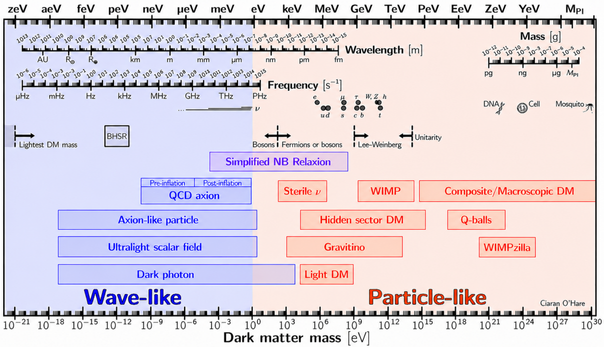


Figure 7: Dark matter and the Simplified Nelson-Barr Relaxion mass ranges.

The Theory

The Simplified NB Relaxion asserts the following sectors:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm roll} + \mathcal{L}_{\rm br} + \mathcal{L}_{\Phi_{\rm N}} + \mathcal{L}_{\rm NB}$$
$$= \mathcal{L}_{\rm SM} + \mathcal{L}_{\phi} + \mathcal{L}_{\rm NB}$$

Relaxion couplings

$$\mathcal{L}_{\rm roll} \supset (\kappa\Lambda_H^2 - \Lambda_H^2 \cos(\frac{\phi}{F}))H^\dagger H + \lambda(H^\dagger H)^2 - r_{\rm roll}^2\Lambda_H^4 \cos(\frac{\phi}{F}),$$

$$\mathcal{L}_{\rm br} \supset -M_{\rm br}^2 H^\dagger H \cos(\frac{\phi}{f}) - r_{\rm br}^2 M_{\rm br}^4 \cos(\frac{\phi}{f}),$$

$$\mathcal{L}_{\Phi_N} \supset \sum_{j=0}^{N}(-m^2|\Phi_j|^2 + g_{\rm clock}^2|\Phi_j|^4) + \Delta V_{\rm clock}(\Phi),$$

The Theory

The Simplified NB Relaxion asserts the following sectors:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm roll} + \mathcal{L}_{\rm br} + \mathcal{L}_{\Phi_{\rm N}} + \mathcal{L}_{\rm NB}$$
$$= \mathcal{L}_{\rm SM} + \mathcal{L}_{\phi} + \mathcal{L}_{\rm NB}$$

Nelson-Barr couplings

$$\mathcal{L}_{\rm NB} \supset \mu_U U\bar{U} + (Y^u)^i{}_j Q_i \tilde{H} \bar{u}^j + y_i^U \Phi_N U \bar{u}^i + \text{H.c.}.$$

$$\mathcal{L}_{\Phi{\rm NB}} \supset \lambda \Phi_N \Phi_N^\dagger H^2 + \lambda_4 \Phi_N \Phi_N \Phi_N^\dagger \Phi_N^\dagger$$

Relaxion Ingredients

Relaxion theory Ingredients [3]

- QCD axion under a new gauge group
- Involves an Inflation Sector
- Axion must extended over a very large field range F
  - $F/f \approx 10^{20}$
- Dynamical Higgs mass (initially at large mass)

Clock work Relaxion [4]

- Considers compact field space F
- N+1 complex scalar fields Φ_j
- Take limit $\epsilon \to 0$ and expand around VEVs $\Phi_j \to U_j \equiv \frac{f}{\sqrt{2}} \mathrm{e}^{i\pi_j/f}$.

$$\mathcal{L}_{\Phi_N} \supset \sum_{j=0}^{N} (-m^2 |\Phi_j|^2 + g_{\text{clock}}^2 |\Phi_j|^4) + \sum_{j=0}^{N-1} (\varepsilon \Phi_j^\dagger \Phi_{j+1}^3 + \text{H.c.}).$$

Relaxion Results

Relaxion theory Result [3]

- Softly broken shift symmetry
- Weak scale becomes technically natural
- Solves the Hierarchy Problem

Clock work Relaxion Result [4]

- Exponentially suppressed decays constant $\frac{F}{f} \approx 3^N$

Relaxion Results

Relaxion cutoff

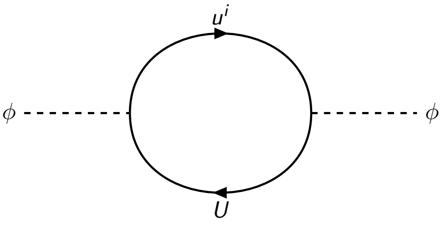


Figure 8: $\phi \to \phi$

$$\Lambda_H \approx \frac{\sqrt{y_i^U y_j^U (Y^{u\dagger} Y^u)_{ij}}}{4\pi} f,$$

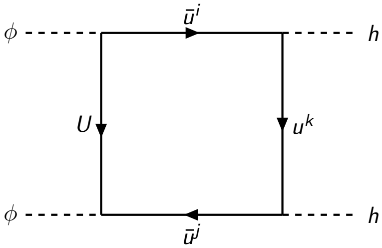


Figure 9: $\phi\phi \to hh$

$$r_{\text{roll}} \approx \frac{4\pi\sqrt{y_k^U y_k^U}}{y_i^U y_j^U (Y^{u\dagger} Y^u)_{ij}}.$$

Relaxion Results

Relaxion mass equations

$$m_\phi^{\mathrm{min}} \approx 1.2 \times 10^{-5}\ \mathrm{eV} \cdot \left(\frac{10^{-2}}{\sqrt{y_i^U y_j^U}}\right)^{5/2} \left(\frac{\sqrt{y_k^U y_k^U}}{10^{-2}}\right)^{3/2} \left(\frac{\Lambda_H}{5\ \mathrm{TeV}}\right)^{5/2}.$$

$$m_\phi^{\mathrm{max}} \approx 86\ \mathrm{MeV} \cdot \left(\frac{\sqrt{y_i^U y_j^U}}{10^{-2}}\right) \left(\frac{5\ \mathrm{TeV}}{\Lambda_H}\right).$$

The Nelson-Barr Sector

The Simplified NB Relaxion asserts the following sectors:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm roll} + \mathcal{L}_{\rm br} + \mathcal{L}_{\Phi_{\rm N}} + \mathcal{L}_{\rm NB} \\ &= \mathcal{L}_{\rm SM} + \mathcal{L}_{\phi} + \mathcal{L}_{\rm NB}\end{aligned}$$

Nelson-Barr couplings

$$\mathcal{L}_{\rm NB} \supset \mu_U U\bar{U} + (Y^u)^i{}_j Q_i \tilde{H}\bar{u}^j + y_i^U \Phi_N U \bar{u}^i + \text{H.c.}.$$

$$\mathcal{L}_{\Phi\rm NB} \supset \lambda \Phi_N \Phi_N^\dagger H^2 + \lambda_4 \Phi_N \Phi_N \Phi_N^\dagger \Phi_N^\dagger$$

Contribution to $\Delta\bar{\theta}$

$$\boxed{\Delta\bar{\theta} = ?}$$

Contributions to $\Delta\bar{\theta}$

$\Delta\bar{\theta}$ calculation

Let M_U be the up-quark mass matrix. $M_U = M_0 + M_1$.

$$M_0 = \begin{pmatrix} (m_u)^i_j & 0 \\ B_i & \mu_U \end{pmatrix} \qquad M_1 = \begin{pmatrix} M_{Q\bar{u}} & M_{Q\bar{U}} \\ M_{U\bar{u}} & M_{U\bar{U}} \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{\mathrm{NB}} &\supset \begin{pmatrix} Q & U \end{pmatrix} M_U \begin{pmatrix} \bar{u} \\ \bar{U} \end{pmatrix} \\ &\supset \begin{pmatrix} Q & U \end{pmatrix} (M_0 + M_1) \begin{pmatrix} \bar{u} \\ \bar{U} \end{pmatrix} \end{aligned}$$

where $(m_u)^i_j = (Y^u)^i_j \frac{v}{\sqrt{2}}$, $B_i = y_i^U \Phi$, and μ_U is the up quark mass term.

Simplification Of $\Delta\bar{\theta}$

$\Delta\bar{\theta}$ calculation

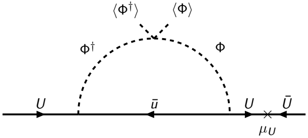
In principle, the $\Delta\bar{\theta}$ angle is given s.t. $\Delta\bar{\theta} = \arg(\det(\mathrm{M}))$

$$
\begin{aligned}
\Delta\bar{\theta} &\simeq \arg(\det(\mathrm{M})) \\
&= \arg(\det(M_0 + M_1)) \\
&= \arg(\det(M_1(M_1^{-1}M_0 + 1)) \\
&= \arg(\det(M_1)\det(M_1^{-1}M_0 + 1))) \\
&\simeq \arg(\det(M_1)(\mathrm{tr}(M_1^{-1}M_0) + 1) \\
&\vdots \\
\Delta\bar{\theta} &\simeq \mathrm{Im}\left(m_u^{-1}\mathcal{M}^{(1)}_{Q\bar{u}} + \frac{1}{\mu_U}\left(\mathcal{M}^{(1)}_{U\bar{U}} - Bm_u^{-1}\mathcal{M}^{(1)}_{Q\bar{U}}\right)\right)
\end{aligned}
$$

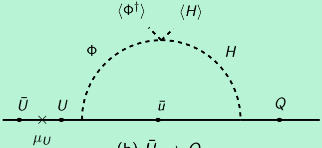
Where $M_1^{-1}M_0 << 1$ [1]

One-loop contributions

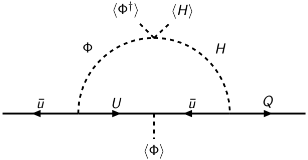
$\Delta\bar{\theta}$ calculation



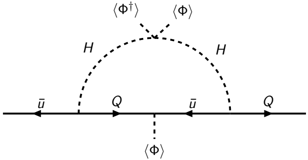
(a) $U \to \bar{U}$



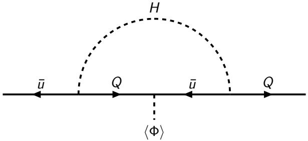
(b) $\bar{U} \to Q$



(c) $\bar{u} \to Q$

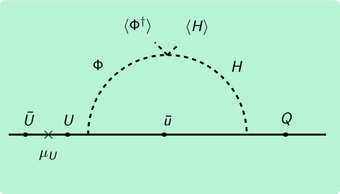


(d) $\bar{u} \to Q$



(e) $\bar{u} \to Q$

$\Delta\bar{\theta}$ calculation

i 

$$\approx i\frac{\mu_U\lambda}{8\sqrt{2}\pi^2 f}y^U_k(m^u)^i{}_k e^{i\theta_N}$$

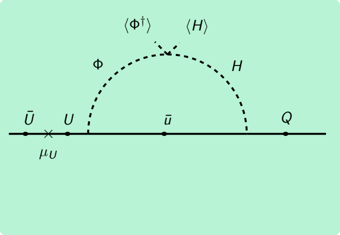
And, using our equation for $\Delta\bar{\theta}$

$$\Delta\bar{\theta} \simeq \mathrm{Im}(\frac{1}{\mu_U}(B_i(m_u^{-1})^i{}_j(\mathcal{M}^{(1)}_{Q\bar{U}})^j)),$$

$$\sim \mathrm{Im}(\frac{1}{\mu_U}(y^U_i\langle\Phi_N\rangle\delta^i{}_k\frac{\mu_U\lambda}{8\sqrt{2}\pi^2 f}y^U_k e^{i\theta_N})),$$

$$\boxed{\Delta\bar{\theta} \sim \frac{\lambda}{16\pi^2}y^U_k y^U_k \sin(\theta_N).}$$

$\Delta\bar{\theta}$ calculation

i 

$$\approx i\frac{\mu_U\lambda}{8\sqrt{2}\pi^2 f}y^U_k(m^u)^i{}_k e^{i\theta_N}$$

And, using our equation for $\Delta\bar{\theta}$

$$\boxed{\Delta\bar{\theta} \sim \frac{\lambda}{16\pi^2}y^U_k y^U_k \sin(\theta_N)}$$

It must be that $\Delta\bar{\theta} \leq 10^{-10}$, so $\lambda \sim \mathcal{O}(10^{-1})$ and $\sin(\theta_N) \sim \mathcal{O}(1)$. Thus,

$$\boxed{y^U \lesssim 10^{-4}}$$

Spurion Anylsis

Dim-5 and Dim-6 operators in the EFT

The spurion analysis gives us the following charge assignments

	$\mathbb{Z}_2$	$U(1)_N$	$U(1)_U$	$U(1)_{\mu_U}$
Φ_N	$-$	-1	0	0
Φ_N^*	$-$	1	0	0
U	$-$	0	1	0
$\bar{U}$	$-$	0	-1	1
y_i^U	$+$	1	-1	0
$(y_i^U)^*$	$+$	-1	1	0
μ_U	$+$	0	0	-1

Table: Charges of the scalar Φ_N, the heavy VLQs U and $\bar{U}$, and the spurions y_i^U and μ_U.

Dim-5

Dim-5 operators in the EFT

The only permitted dim-5 operators are

$$\frac{\mu_U}{(4\pi f)^2} y_i^U y_j^U \alpha^{ij} \Phi_N^2 U \bar{U} \quad \text{and} \quad \frac{\mu_U}{(4\pi f)^2} \beta^{ij} (y_j^U)^* \Phi_N^* Q_i \tilde{H} \bar{U}$$

Their contributions give

$$\Delta\bar{\theta} = \frac{\alpha^{ij}}{32\pi^2} y_i^U y_j^U \sin 2\theta_N. \tag{3}$$

$$\Delta\bar{\theta} \sim \frac{1}{16\pi^2} y_i^U (Y_u^{-1})^i{}_j \beta^{kj} y_k^U \sin\theta_N \tag{4}$$

Under the same analysis, $|y^U| \lesssim 10^{-4}$

Dim-6

Dim-6 operators in the EFT

The only permitted dim-6 operator is

$$\frac{\gamma^{ijk}{}_{l}}{(4\pi f)^2}(y_i^U y_j^U)\Phi_N^2 Q_k \tilde{H}\bar{u}^l. \quad (5)$$

It's contribution give

$$\Delta\bar{\theta} \sim \frac{(Y_u^{-1})^l{}_k \gamma^{ijk}{}_l}{32\pi^2}(y_i^U y_j^U)\sin(2\theta_N). \quad (6)$$

But now, $|y^U| \lesssim 10^{-3}$

Results | Parameter Space | $|y_i^U| \lesssim 10^{-4}$

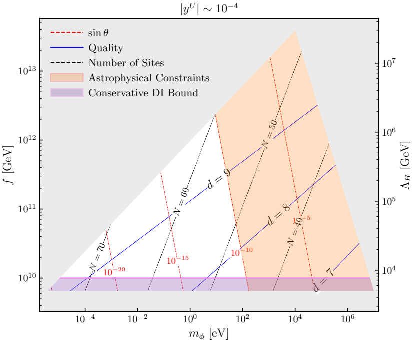


Figure 9: Conservative result

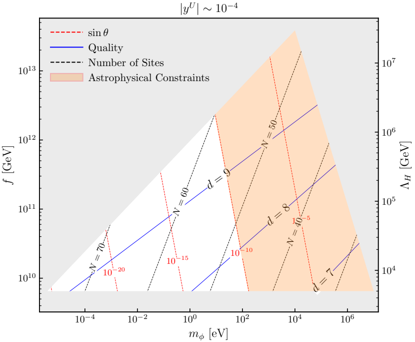


Figure 10: Optimistic result

Results | Parameter Space | $|y_i^U| \lesssim 10^{-2}$ $|D - Type$

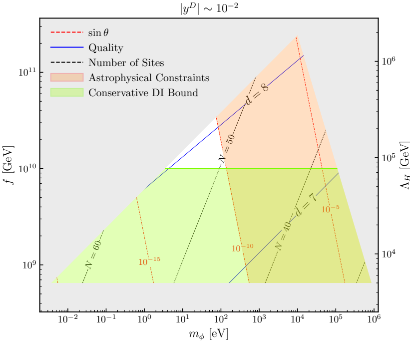


Figure 11: Conservative result

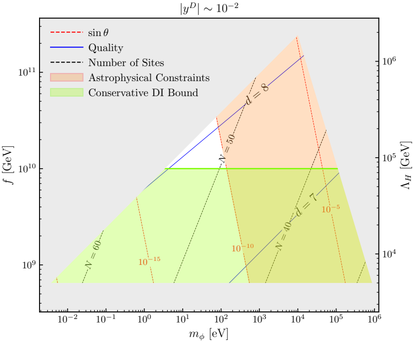


Figure 12: Optimistic result

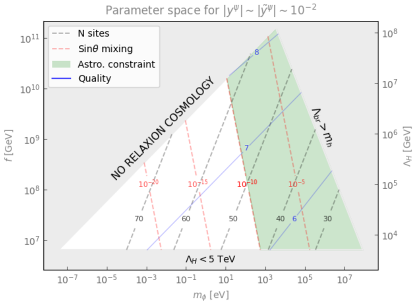


Figure 12: The Nelson-Barr Relaxion

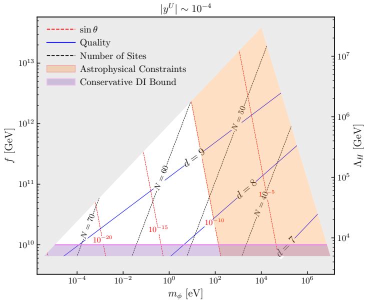


Figure 13: The Simplified Nelson-Barr Relaxion

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